SUPERPOSED TEMPERAMENTS: A NEW APPROACH TO AMPLIFY THE INTERPLAY BETWEEN JUST-TEMPERED AND EQUAL-TEMPERED SYSTEMS

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1.1 INTRODUCTION

Issues in tuning have always existed within music. regardless of the tuning system one assigns themselves to, there will eventually be some limitation. These limitations come in two distinct forms:

a). Intonation is prioritized, but intervallic (harmonic) function is limited

b). Intervallic function is prioritized, intonation is limited

Prior to the standardization of equal-tempered systems, Western music was required, out of necessity, to adhere to Category A. Pythagorean tuning took precedence, while the Renaissance Era displayed more experimental solutions through Zarlino's approximated meantone temperament (1558) and Salinas's Just Intonation system (1577)[1]. Since, a slow progression toward an equal temperament can be observed. From the first logarithmic calculations of 12-tone equal temperament in 1630 by German engineer Johann Faulhaber[2], through the strange and spectacular *well*-temperaments and concentric tunings of Andreas Werckmeister[3], and the inevitable smoothing of unequal temperaments by Johann Neidhardt[4], we arrived somewhere in the mid-1700s with a thorough and agreed upon equal temperament of twelve notes to the

[1] Rasch, Rudolf. "Tuning and Temperament." Chapter 7. In *The Cambridge History of Western Music Theory*, edited by Thomas Christensen, 193–222. Cambridge University Press, 2002.
[2] Ibid.
[3] Ibid.
[4] Ibid.

octave. And since Bach, up until arguably the spectralists of the 20th century, Western music has almost entirely fallen into Category B.

Up until the latter part of the twentieth century, it seemed as if just-intonation (JI) and equal temperament (ET) were a dichotomy that could not be mixed. We have observed, particularly through the meantone temperaments of Werckmeister, that living on a spectrum between the two often yields challenging results, and that radically increasing the amount of equally tempered intervals between octaves was needlessly complex for performers.

However, today we live in a world of electronics and of machine-level precision. We can create tones of all kinds, calculating waveforms at complex ratios and proportions. JI need not fail us as we have learned to tonicize and modulate to various different keys, partials, and spectra. It seems that, if we are to search for harmonic clarity, JI finds its rightful place yet again. However, 12-tone equal temperament (12-TET) still stands tall. Discoveries are being made and organizational systems are being invented through studying advanced concepts in set theory[5], geometry[6], and interval transformations[7], proving that we are still finding new methods of expression within twelve equal divisions of the octave. But now a quarter through the 21st century, we still seem to only find music that is complacent with either Category A or Category B.

The term *superposition*, as used in film, physics, and geometry, is "the action of causing two or more sets of a physical phenomena to coincide, or coexist in the same place; the fact or an

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[5] Straus, Joseph. "Motive, Voice Leading, and Harmony." Chapter 4. *An Introduction to Post-Tonal Theory: Fourth Edition*, 159-198. W. W. Norton & Company, 2016.

[6] Tymozcko, Dmitri. *A Geometry of Music: Harmony and Counterpoint in the Extended Common Practice*. Oxford University Press (2010).
Also: Lewin, David. "Some Compositional Uses of Projective Geometry." *Perspectives of New Music* 42, no. 2 (2004): 12–63. http://www.jstor.org/stable/25164553.

[7] Klumpenhouwer, Henry. "Interval." Chapter 2. *The Oxford Handbook of Critical Concepts in Music Theory*, ed. Alexander Rehding and Steven Rings.

instance of such coexistence" as defined by the Oxford English Dictionary[8]. This paper explores the superposition of an equal tempered system against a system rooted in just intonation, and the acoustic ramifications for such a method. The purpose of doing this is to explore the intervals and scales that are created within, analyzing their sonorities, and finding a way to deliberately pass between Category A and Category B at will and as many times as is desired over the course of a piece of music. Both categories fuse, providing enough differences to obtain strong dissonances between intervals, yet cohere closely enough where functional harmony, key relationships, and modulation all play an important role, creating a new system of sound organization within.

2.1 DERIVING SUPERPOSED SYSTEMS: DERIVING EQUAL TEMPERAMENTS

First, we must decide on which scales to superpose. We must choose one JI scale and one ET scale to combine, and there must be no difference in the amount of pitch classes when superposing. If we were to take, for example, a 7-tone equal-tempered (7-TET) scale and combine it with the first four prime partials of the harmonic series, that would create an unequal superposition (7 equal-tempered pitch classes vs. 4 just-tuned pitch classes), therefore giving precedence to equal temperament over just intonation. Our goal is to create a sound world where both Categories are emphasized equally, so that a perfect mixture of the two is consistent, though it may yield fascinating results to analyze the effects of unequal derivations of Categories in a future study of wider scope.

Equal temperament of any value can be derived utilizing the following process:

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[8] Oxford English Dictionary, "superposition (n.)," December 2025, https://doi.org/10.1093/OED/8493013173.

$\sqrt[x]{2}$ where *x* = the total number of pitch classes to make an octave.

If we are to derive the standard 12-TET in Western music, we would use $\sqrt[12]{2}$. While this yields an irrational number (therefore explicitly separating it from rational music, i.e., from JI), its approximate value is 1.05946. This number represents how much to multiply a given frequency by in order to increase the pitch by one 12-TET half step. So, if we take a concert A sounding at 440hz and multiply it by 1.05946, we get an A-sharp sounding at 466.1624hz. If we want to find the next half-step up, we take 1.05946 and multiply it by itself, yielding 1.12246, then multiplying this by our concert A 440hz, producing a concert B 493.9hz. Every time we multiply our initial product (in this case, 1.05946) by our newest product (1.12246), we find the next pitch in our scale. *Ex. 1* shows all 12-TET pitches and their respective multipliers:

Ex. 1

Multiplier	**Frequency (440 *times* the multiplier)**	**Corresponding 12-TET pitch**	**Interval**
1	440 (440 *times* 1)	A	u
1.0595	466.18 (440 *times* 1.0595)	A sharp	m2
1.1225	493.9	B	M2
1.1892	523.248	C	m3
1.2599	554.356	C sharp	M3
1.3348	587.312	D	P4
1.4142	622.248	D sharp	d5
1.4983	659.252	E	P5
1.5874	698.456	F	m6
1.6818	739.992	F sharp	M6
1.7818	783.992	G	m7
1.8877	830.588	G sharp	M7
2	880	A	O

2.2 DERIVING SUPERPOSED SYSTEMS: DERIVING JUST-TUNED SCALES

Just intonation is derived from ratios. If the fundamental pitch of a given scale is A0 (27.5 hz), then 27.5hz *times* 2 (2:1 ratio) is the second partial of the harmonic series, or A1 at 55hz. Continuing up the series, 27.5 *times* 3 (3:1 ratio to the fundamental) is the third partial of the harmonic series, or E1 at 82.5hz. When deriving a scale from JI, it is crucial to note that every *prime* partial except for 2 yields a new pitch class. Every partial in the harmonic series that is a prime number over the fundamental, e.g. 3, 5, 7, 11, 13... will produce a new pitch because there is no way to simplify their ratios. In other words, because 4 is divisible by 2, and therefore divisible by 1, the 4th partial will produce another concert A. Because 9 is divisible by 3, the 9th partial will produce a concert E. The first twelve new partials in the harmonic series are listed in *Ex. 2*:

Ex. 2

Partial	**Corresponding pitch accurate to the nearest 12-TET cent**	**Approximate interval**
1:1	A (+0)	u
1:3	E (+2)	P5 (wide)
1:5	C sharp (-14)	M3 (narrow)
1:7	G (-31)	m7 (narrow)
1:11	D sharp (-49)	d5 (narrow)
1:13	F (+41)	m6 (wide)
1:17	A sharp (+5)	m2 (wide)
1:19	C (-2)	m3 (narrow)
1:23	D sharp (+28)	d5 (wide)
1:29	G (+30)	m7 (wide)
1:31	G sharp (+45)	M7 (wide)
1:37	C (-49)	m3 (narrow)

There are a few issues with this chart. First of all, as the harmonic series increases, the relation of pitches to the fundamental tone becomes less recognizable to the human ear. By the time we achieve somewhere around a 23rd partial, our ears cannot decipher any consonant relation to the fundamental pitch, because it is so faint. Therefore, a 12-tone JI scale might not make much audible sense.

Furthermore, there are some “new” pitch classes at lower partials in the harmonic series that are derived from adding multiples other than the fundamental. For example, the 3rd partial (3:2 ratio) of concert A (an E +2 cents, referred to as E^{+2}) can be inverted, reflecting the ratio over the “A” axis to obtain a concert D (-2 cents, referred to as D^{-2}). Also, the 9th partial is a 9:8 ratio. Both fractions can be reduced, but there is no common factor between the two, and it yields a new pitch class, a B^{+4}. The same is true for the 15th partial, where the 16:15 ratio has no common factors, yielding the pitch class $G\#^{-12}$. By minutely manipulating the ratios while still staying true to their function, a scale of 8 pitches can be created from JI utilizing the first 16 partials of the harmonic series:

Ex. 3

Scale degree	**Partial**	**Pitch class**
1	1	A^{+0}
2	9	B^{+4}
3	5	$C\#^{-14}$
4	4	D^{-2}
5	11	$D\#^{-49}$
6	3	E^{+2}
7	13	F^{+41}
8	7	G^{-31}
9	15	$G\#^{-12}$
10	2	A^{+0}

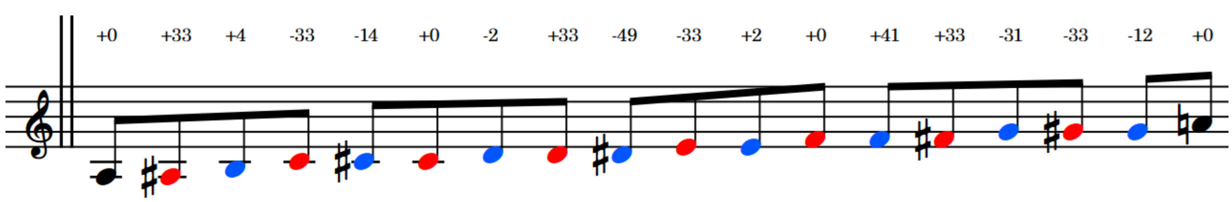
All of these pitches, including partials 3, 5, 7, 11, and 13 are all audibly discerned as consonances related to their fundamental pitch, concert A.

Creating a JI scale larger than this could cause some logistical problems, as pitches become further and further removed from the fundamental. And since we must create a JI scale with the same number of pitch classes as our ET scale, it seems like our maximum ET scale will not be able to have more than approximately eight or nine pitch classes. This is not an issue though, because, as is evident in the next section of the paper, these scales will become increasingly complex as new pitches are added.

2.3 DERIVING SUPERPOSED SYSTEMS: SUPERPOSING JI AND ET

Let us take the JI scale created in *Ex. 3* (from nine different pitch classes) and superpose a 9-TET scale where they both share the same fundamental concert A, referred to as a *shared value*, creating the following 17-note *superposed* scale:

Ex. 4



Red noteheads belong to the $A^{9\text{-TET}}$ scale and the blue notes belong to the A^{JI} 15-limit scale ("15-limit" meaning we are *limiting* our just intonation at the fifteenth partial).

Superposed scales can be created in this way using any number of pitch classes. But for a superposed scale to share an equal amount of ET and JI pitches, it must have an *odd* number of unique pitch classes: (ET pitch classes) + (JI pitch classes) – 1 (pitch shared between the two) = an odd number.

3.1 THE SUPERPOSED PENTATONIC SCALE: DERIVATION

The remainder of this paper analyzes the superposed system that is created when superposing the 5-TET scale onto the 7-limit JI scale, more commonly known as the pentatonic scale with just tuning. This superposed system is useful to analyze for a number of reasons. For one, pentatonic scales occur naturally in so many different cultures and within so many different genres of music. Contrarily, the 5-TET scale is unusual to us, primarily due to the fact that it shares no pitches close to in common with pentatonic or our traditional 12-TET systems. Combining the consonance-dominating pentatonic scale with that of the unusual 5-TET scale will amplify their distinct features and will serve as a great tool to study the relationships that superposed systems can provide.

For this paper, we will be superposing the $C^{5\text{-}TET}$ scale upon the C^{JI} 7-limit scale:

Ex. 5

Scale degree	Pitch class	JI function	ET function
1	C (+0)	Shared value, fundamental	Shared value
2	D (+4)	9th partial	n/a
3	D (+40)	n/a	Scale degree 2 in 5-TET
4	F (-20)	n/a	Scale degree 3 in 5-TET
5	F (-2)	3rd partial, inverted	n/a

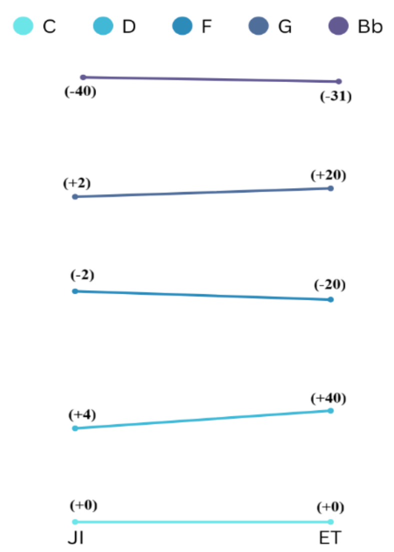
6	G (+2)	3rd partial	n/a
7	G (+20)	n/a	Scale degree 4 in 5-TET
8	B flat (-40)	n/a	Scale degree 5 in 5-TET
9	B flat (-31)	7th partial	n/a

Ex. 5 shows the superposed scale that we will be working with, as well as the functions of each of the nine scale degrees within.

3.1 THE SUPERPOSED PENTATONIC SCALE: SIMPLE MODULATIONS

We can derive a functional method of sound organization from the sonorities that this system produces. First, we will split the superposed system back into its two constituents and map out the voice leading changes when shifting from the JI pitches to the ET pitches of the scale:

Ex. 6



Pitch class	**JI value**	**ET value**	**Difference (| JI value – ET value |)**
C	0	0	0
D	+4	+40	36
F	-2	-20	18
G	+2	+20	18
B flat	-40	-31	9

As is made clear by the table in *Ex. 6*, the differences between these values are always being altered by a factor of 9 cents (9, 18, 27, 36, etc). This pattern is significant and is axiomatic of the relationship between the two scales. As we develop our superposed system, these differences will always offset at a factor of 9 cents, proving useful for us later.

Now, we can consider shifting pitch areas, or possibly even changing keys. First, we will build a set of four additional 5-TET scales whose *shared value* (see 2.3) becomes one of the five partials in our C^{JI} scale:

Ex. 7

Pitch class from C-JI Pentatonic (with cent deviation)	**List of pitch classes within 5-TET scale (with cent deviation)**
C^{0}	C^{0}: D^{+4}, F^{-20}, G^{+20}, Bb^{-40}
D^{+4}	D^{+4}: F^{-56}, G^{-16}, Bb^{-76}, C^{-36}
F-2	F^{-2}: G^{+38}, Bb^{-22}, C^{+18}, D^{+58}
G+2	G^{+2}: Bb^{-58}, C^{-18}, D^{+22}, F^{-38}
Bb-31	Bb^{-31}: C^{+}), D^{+49}, F^{-11}, G^{+29}

By applying the same analysis of *Ex. 6* to the 5-TET scales of *Ex. 7*, we will find that, again, all differences between JI values and ET values for each pitch class are offset by some factor of 9 cents. Differences are as follows:

Ex. 8

5-TET scale (cent dev.)	**5-TET to C-JI Pentatonic motion**	**Difference in cents**
C^{+0}	C^{+0} to C^{+0}	0 (cents)
	D^{+40} to D^{+4}	36
	F^{-20} to F^{-2}	18
	G^{+20} to G^{+2}	18
	Bb^{-40} to Bb^{-31}	9
D^{+4}	D^{+4} to D^{+4}	0 (cents)
	F^{-56} to F^{-2}	54
	G^{-16} to G^{+2}	18
	Bb^{-76} to Bb^{-31}	45
	C^{-36} to C^{+0}	36
F^{-2}	F-2 to F-2	0 (cents)
	G+38 to G+2	36

(continued)

	Bb-22 to Bb-31	9
	C^{+18} to C+0	18
	D^{+58} to D^{+4}	54
G^{+2}	G^{+2} to G^{+2}	0 (cents)
	Bb^{-58} to Bb^{-31}	27
	C^{-18} to C^{+0}	18
	D^{+22} to D^{+4}	18
	F^{-38} to F^{-2}	36
Bb^{-31}	Bb^{-31} to Bb^{-31}	0 (cents)
	C^{+9} to C^{+0}	9
	D^{+49} to D^{+4}	45
	F^{-11} to F^{-2}	9
	G^{+29} to G^{+2}	27

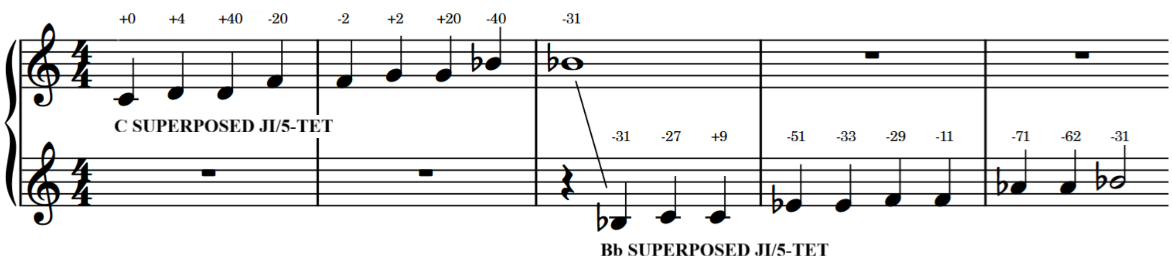
With the differences now listed, we can simplify them by a factor of 9. Then, we can add them all together and assign them a cumulative value. A larger value signifies a more distantly related modulation *from* C^{JI}:

Ex. 9

5-TET scale	**List, in scale order, of differences from *Ex. 8***	**Simplification by a factor of 9**	**Column 3 cumulative value**
C^{+0}	0, 36, 18, 18, 9	0, 4, 2, 2, 1	**9**
D^{+4}	0, 54, 18, 45, 36	0, 6, 2, 5, 4	**17**
F^{-2}	0, 36, 9, 18, 54	0, 4, 1, 2, 6	**13**
G (+2)	0, 27, 18, 18, 36	0, 3, 2, 2, 4	**11**
Bb (-31)	0, 9, 45, 9, 27	0, 1, 5, 1, 3	**10**

Based on this data, if we want to create a simple *common-tone* modulation to pivot from one scale to another, C^{JI} to C^{5TET} is the closest, with a cumulative valued pitch-shift of 9. This particular modulation doesn't do much for us, since our superposition of C^{JI} and C^{5TET} remains the same. However, a common-tone modulation to $Bb^{-31:5TET}$ has a cumulative pitch-shift of 10, which is only 9 cents more incumbent than our original superposed scale. Bb^{-31} would become the new tonic to which we could modulate and build our new superposed scale. *Ex. 10* displays this type of modulation, referred to as a *simple modulation*:

Ex. 10



This motion can be achieved similarly upon the other 5-TET scales, however based on our cumulative values from *Ex. 9*, Bb^{-31} is the closest related key, followed by G^{+2}, F^{-2}, and D^{+4} being the most distantly related key.

When exploring the same model but inversely building four additional JI scales upon our C^{5TET} scale, we can assess which key is most related:

Ex. 11

Pitch class from C-5TET	**List of pitches within JI scale**	**5-TET to C-JI Pentatonic motion | Difference in cents**
C^{+0}	C^{+0}: D^{+4}, F^{-2}, G^{+2}, Bb^{-31}	C^{+0} to C^{+0} = 0 (cents) D^{+40} to D^{+4} = 36 F^{-20} to F^{-2} = 18 G^{+20} to G^{+2} = 18 Bb^{-40} to Bb^{-31} = 9
D^{+40}	D^{+40}: F^{-56}, G^{+38}, Bb^{-58}, C^{+9}	D^{+40} to D^{+40} = 0 F^{-20} to F^{-56} = 36 G^{+20} to G^{+38} = 18 Bb^{-40} to Bb^{-58} = 18 C^{+0} to C^{+9} = 9
F^{-20}	F^{-20}: G^{-16}, Bb^{-22}, C^{-18}, D^{+49}	F^{-20} to F^{-20} = 0 G^{+20} to G^{-16} = 36 Bb^{-40} to Bb^{-22} = 18 C^{+0} to C^{-18} = 18 D^{+40} to D^{+49} = 9

(continued)

G^{+20}	G^{+20}: Bb^{-76}, C^{+18}, D^{+22}, F^{-11}	G^{+20} to G^{+20} = 0 Bb^{-40} to Bb^{-76} = 36 C^{+0} to C^{+18} = 18 D^{+40} to D^{+22} = 18 F^{-20} to F^{-11} = 9
Bb^{-40}	Bb^{-40}: C^{-36}, D^{+58}, F^{-38}, G^{+29}	Bb^{-40} to Bb^{-40} = 0 C^{+0} to C^{-36} = 36 D^{+40} to D^{+58} = 18 F^{-20} to F^{-38} = 18 G^{+20} to G^{+29} = 9

As we can see from *Ex. 11*, the change in cents for each pitch class follows the same pitch shift: 0 cent shift for the first scale degree, 36 cent shift for the second, 18 cent shift for the third, 18 cent shift for the fourth, and 9 cent shift for the fifth. When dividing by a factor of 9 and adding their cumulative displacements, we arrive at the same cumulative value as the *lowest* 5-TET cumulative value from before: 9. This means that setting a new fundamental tone upon any 5-TET pitch class within the superposed scale is an equidistant simple modulation in terms of quality. It is also a higher quality simple modulation than *all* modulations from a JI pitch class to a 5-TET scale (like the one present in *Ex. 10*).

3.2 THE SUPERPOSED PENTATONIC SCALE: 25-TONE PARENT SCALE

All pitches present in *Ex. 8* are also present in *Ex. 11*. From them, we can create the following 25-note scale between one octave, within which we can employ our simple modulations freely:

Ex. 12

Scale degree	**Pitch (in cents)**	**Interval from previous (in cents)**
1	C^{+0}	18
2	C^{+9}	9
3	C^{+18}	9
4	D^{+4}	186
5	D^{+22}	18
6	D^{+40}	18
7	D^{+49}	9
8	D^{+58}	9
9	F^{-56}	186
10	F^{-38}	18
11	F^{-20}	18
12	F^{-11}	9
13	F^{-2}	9
14	G^{-16}	186
15	G^{+2}	18
16	G^{+20}	18
17	G^{+29}	9
18	G^{+38}	9
19	Bb^{-76}	186
20	Bb^{-58}	18
21	Bb^{-40}	18
22	Bb^{-31}	9
23	Bb^{-22}	9
24	C^{-36}	186
25	C^{-18}	18

We now have a *parent* scale of 25 tones. There are at least 5 keys with which we can modulate in and out of. We can also call intervals of 9 cents *semitones*, intervals of 18 cents *whole tones*, and the most interesting intervals present, those of 186 cents, can be called *macro tones*.

3.3 THE SUPERPOSED PENTATONIC SCALE: APPROXIMATE MODULATIONS TO NEW KEYS

What is most notable about this 25-tone system is the massive 186 cent leap. If we take the pitch class on either side of any of these 186 cent leaps (for example, C^{+18} and D^{+4}, we find a sonority closely related to a different partial in a *new* harmonic series. Let D^{+4} be the 10^{th} partial of a new harmonic series, and C^{18} be the 9^{th} partial. These two tones exhibit a 4-cent comma deviation from a Bb^{+14} or Bb^{+18} JI scale. If we split the difference, a Bb^{+16} JI scale is created with only a 2-cent comma, practically indistinguishable to the human ear.

Therefore, this entire 25-tone scale is capable of modulating on its own to any of its five *neighboring* keys: $Bb^{+16:JI}$, $C^{+28:JI}$, $Eb^{-32:JI}$, $F^{+8:JI}$, and $G^{+48:JI}$.

And, if all of these new keys to our comparative 12-TET appear terrifyingly complicated, consider the following:

a). Take the 23rd and 24th scale degree from *Ex. 12* and use this dyad to modulate to G^{+50} instead of G^{+48}, maintaining the 4-cent comma instead of splitting the difference.

b). G^{+50} is 1 cent off from the 11th partial of D flat. Tonicize D-flat from your G^{+50} modulation.

c). A successful modulation from $C^{JI/5TET}$ to D flat has been achieved utilizing only five cents of deviation from true intonation, split across two modulatory motions.

The following additional modulatory motions can be derived from this 25-tone scale with commas below five cents:

1. $C^{5\text{-}TET}$ includes a D^{+40}, which can be considered the 13th partial of Gb with a deviation of 1 cent. This creates a C to Gb modulation.
2. $F^{-2:5\text{-}TET}$ (3rd scale degree of C^{JI}) includes a G^{+38}, which can be considered the 13th partial of B with a deviation of 2 cents.
3. $Bb^{-31:5\text{-}TET}$ (5th scale degree of C^{JI}) includes a D^{+49}, which can be considered the 11th partial of A with a deviation of 2 cents. This creates a C to A modulation.
4. $D^{+4:5\text{-}TET}$ (2nd scale degree of C^{JI}) includes a G^{-16}, which can be considered the 5th partial of Eb with a deviation of 2 cents. This creates a C to Eb modulation.

Repeat this motion as many times as you'd like to achieve *approximate modulations* to all 12-TET keys.

3.4 THE SUPERPOSED PENTATONIC SCALE: TRUE MODULATIONS TO NEW KEYS

A *true modulation* is one where there is no comma when pivoting upon a given pitch class. In fact, a true modulation does not mathematically exist when superposing tuning systems that are derived irrationally (all ET scales are inherently irrational, as they are calculated using the square root of 2). However, getting within one cent of deviation is a useful practice because most modern MIDI-programmed systems are capable of pitch-bending with accuracy to the nearest cent. Similarly, less than one cent of deviation is pretty darn close to perfect accuracy, and if found within our superposed 5-TET/7-limit JI system, we can modulate wherever we would like freely and without the need to "reset" at different key areas.

Finding less than one cent of deviation can be achieved through numerous common-tone modulations between different keys, continuously searching for a pitch that doubles as a partial of a different harmonic series.

The methodology behind achieving this type of modulation is as follows:

a). Divide your superposed scale into the two fundamental systems, as has been done previously

b). Choose one pitch class from your first system. This now becomes the tonic/fundamental of your second system.

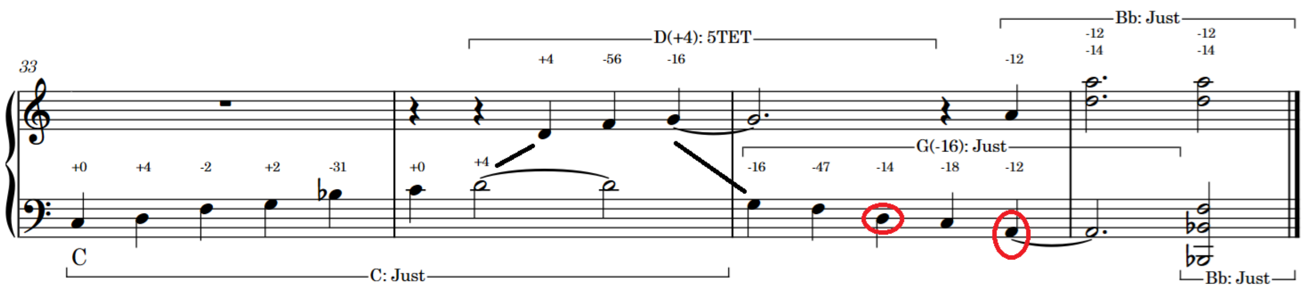
c). Apply step b). to your new scale, switching back now to your first system.

d). Repeat this process, searching for pitch classes tuned to the following cents: (-14, -31, -49, +41, -12). These are the same tunings as audible partials in a given harmonic series.

Often, these partials are merely the partials of the original key's harmonic series, but not always. Once you have successfully discovered a pitch class related to a new fundamental, modulate to that new fundamental key and rebuild your superposed system atop it.

Ex. 13 displays how this modulation is achieved:

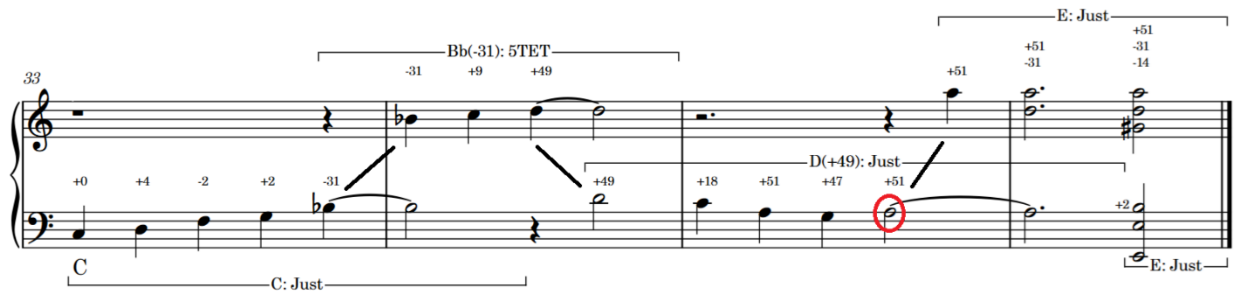
Ex 13: C^{JI} to Bb^{JI} Modulation utilizing 5TET and JI scales with less than 1 cent of deviation



In *Ex. 13*, we begin in C^{JI}, then we use three common tone modulations in succession to the following keys: $D^{+4:5TET}$, $G^{-16:JI}$, and finally Bb^{JI}. Take close note that, for this example, there are actually two pitch classes in common with Bb^{JI} that are part of the G^{-16}JI scale: D^{-14} and A^{-12}, both acting as the 5th and 15th partials respectively. This motion creates a slow and kaleidoscopic progression down exactly 200 cents, or one perfect 12-TET whole step, effectively giving us access to modulate to an entire 12-TET whole tone scale, if we so choose.

There are other forms of *Ex. 13* that can be found through creating these complex systems. However, this system is not infinite. Modulating too many times to distantly related keys will eventually lead one back to where one started. In fact, it usually only takes 3-5 degrees of separation until one finds themselves re-analyzing the same $C^{JI/5TET}$ scale which began the whole process. *Ex. 13* displayed how one can navigate from C^{JI} to Bb^{JI}, but one is, in fact, only able to navigate to one other key: E^{JI} as shown in *Ex. 14*:

Ex. 14: C^{JI} to E^{JI} modulation utilizing 5TET and JI scales



Therefore, there are two distinct groups of JI/5TET superposed systems that encompass an entire octave: one assigned to each whole tone scale, all with their own matrices with which to modulate.

To summarize, from the creation of the superposed $C^{JI/5TET}$ 9-note scale, we have achieved the following:

1. Created simple *common tone* modulations with which we can pivot to nearby keys
2. Created a 25-note scale with which we can modulate freely at all parts of a 12-TET octave with a comma of less than five cents.
3. Created, from our 25-note scale, a matrix of common-tone modulations that aid us in modulating away from our parent $C^{JI/5TET}$ system to all keys of the traditional C whole tone scale.

4.1 DISCUSSION

Now that we have discovered appropriate ways to modulate within our system, we can create music that utilizes traditional harmonic devices with entirely new pitches. Taking a highly consonant scale (just-tuned pentatonic scale) and superposing a highly dissonant one (5-TET scale) atop it provides us with a complex and colorful array of harmonies with which to track musical intensity and artistic expression. If utilizing a superposed scale such as this in an artistic work, there is an implicit uneasiness within the music, void of the blaring dissonances present in more dodecaphonic or experimental music within the 12-TET system. Instead, it is fuzzy, anxious, and faded. The pentatonic scale can shine brilliantly, then sink into the blurry 5-TET daze.

Intervallic relationships can also be expanded upon within superposed scales, creating a complex network of new interval classes and more possibilities for sound organization. And need us not limit ourselves to just this one superposed scale! Superposition between just intonation and equal temperament gives us the flexibility to delve into even more systematic pitch organization. We can create a whole matrix of superposed scales and note each one's physical characteristics, intervallic makeup, and geometric structure as we have with this scale. We can go further and analyze the power of serialism and set theory, or go through modes in a search for tonic polarity. There is a myriad of ways to approach JI/ET superposition, though these might be outside of the scope of this paper: which is to prove that working in a framework of tuning/temperament superposition is not only possible, but filled with musical potential. Of course, live performance of these complex modes might be hindered. However, with the power of electroacoustic music; of MIDI pitch-bends for ET and natural resonances taken from harmonic-series-based instruments, there are so many directions this kind of music can take, propelling us further forward in the pursuit of the new, the experimental, and the voice of the present.

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